Nonequilibrium fluctuation-induced phase transport in Josephson junctions

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We show that nonequilibrium current fluctuations of various types can give rise to *net* voltages in superconducting tunnel junctions. This is the result of a fluctuation-induced net rate of change of the phase difference of the superconducting order parameter across the junction. Various exact expressions are derived for the mean voltage and are evaluated explicitly in certain limits, as well as by numerical simulations. We show that these phenomena are due to an asymmetry in the spectral properties of the current noise, which should be a fairly ubiquitous feature of nonequilibrium fluctuations in nature.

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In the past year or so there has been a flurry of interest in fluctuation-induced transport phenomena [1], focused on the observation that nonequilibrium fluctuations can lead to transport in the presence of a spatial asymmetry [2]. It has also been pointed out that mean-zero noise of a more complicated asymmetric type can lead to similar phenomena even in the absence of a spatial asymmetry [3]. Preliminary attempts have been made to apply these phenomena to the operation of biomolecular motors [4], as applications of new molecular separation techniques [5], to condensed matter type systems [6,7], and to understanding the kinetics of single ion channels [8]. All of this recent work has focused on the transport of particles along a given spatial axis. Here we show that these types of phenomena can appear in more general contexts by demonstrating that the "transport" of the phase difference of a quantum mechanical order parameter via an analogous process leads to fluctuation-induced phenomena in superconducting electronic devices.

Most of the previous work focused on completly symmetric nonequilibrium noise in the presence of a spatial asymmetry. In the present case a complete symmetry of the underlying system (antisymmetry of the pair current across the junction as a function of the phase difference) is imposed by time reversal invariance, and the effect is due to a spectral asymmetry in the noise which we call *dynamical asymmetry* to distinguish it from spatial asymmetry.

A typical superconducting (Josephson) junction consists of a junction shunted by a resistance R, and driven by a current $I(t)$ as pictured in Fig. 1. Such junctions are of interest not only in terms of their technological applications, but because of their ability to exhibit (singly and in arrays) a number of important types of nonlinear phenomena, including phase locking, bifurcations, chaos, solitonic excitations, and pattern formation [9]. Josephson showed that electron pairs could tunnel through a narrow insulating material between two superconductors [10]. The pair current across the junction is given by $J_p = J_c \sin\phi$, where ϕ is the phase difference of the superconducting order parameter across the junction and J_c is a critical current. The evolution of the phase difference of the current carrying state is described by the equation

$$\frac{\hbar}{2eR}\dot{\phi} + J_c \sin\phi = I(t), \tag{1}$$

where $I(t)$ is a driving current [11]. Here we will be interested in totally unbiased driving $\langle I(t)\rangle = 0$.

Of particular interest is the voltage $V(t)$ across the circuit. The phase difference is related to the voltage according to

$$\phi(t) - \phi(0) = \frac{2e}{\hbar}\int_0^t V(s)\, ds \tag{2}$$

so that $V(t) = (2e/\hbar)\dot{\phi}$. We will show that the right conditions of the nonequilibrium fluctuations of $\eta(t)$ can lead to a fluctuation-induced net rate of change of the phase difference across the junction, and consequently a fluctuation-induced net voltage $\langle V(t)\rangle = (2e/\hbar)\langle\dot{\phi}\rangle$ where the $\langle\,\rangle$ indicate time averages.

As a first example where the mechanism should be readily grasped by the general reader we will consider the rather elementary case of "slow noise." We set $I(t) = \xi(t) + f(t)$, where $\xi(t)$ is "fast" Gaussian white noise (either thermal or externally applied) $\langle\xi(t)\rangle = 0$, and $\langle\xi(t)\xi(s)\rangle = 2D\delta(t-s)$ and the driving $f(t)$ is "slow noise" with mean zero $\langle f(t)\rangle = 0$. In pratice any net fluctuating forces are usually included in the determinis-

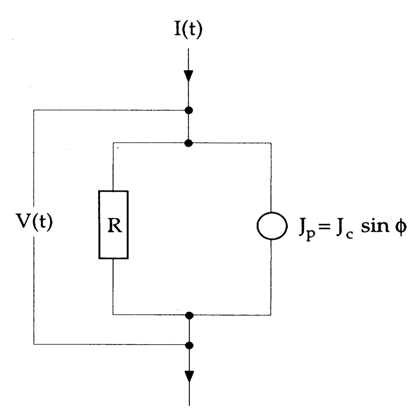

FIG. 1. Resistively shunted Josephson junction.

tic parts of the equation. Additionally, any net current can be canceled by an externally applied current so that $\langle f(t)\rangle = 0$. Equation (2) takes the form

$$\dot{\phi} + \omega_j \sin(\phi) = \zeta(t) + \eta(t), \tag{3}$$

where $\langle \zeta(t)\zeta(s)\rangle = 2\tilde{D}\delta(t-s)$ with $\tilde{D} = (4e^2R^2/\hbar^2)D$, $\eta(t) = 2eRf(t)/\hbar$, and $\omega_j = 2eRJ_c/\hbar$. The evolution of the probability density for ϕ is given by the associated Fokker-Planck equation

$$\partial_t \rho(\phi) = \partial_\phi \left[\Psi(\phi,t) + \tilde{D}\partial_\phi\right]\rho(\phi)\ , \tag{4}$$

where $\Psi(\phi,t) = -\omega_j \cos\phi - \eta(t)\phi$. The steady-state solution (for constant η) is found by imposing periodic boundary conditions $\rho_s(\phi) = \rho_s(\phi + 2\pi)$, and normalization $\int_0^{2\phi} \rho_s(x)\, dx = 1$ [12]. This yields an exact expression for the mean rate of change of ϕ

$$\left\langle \dot{\phi} \right\rangle = \frac{2\pi[1-\exp(-2\pi\eta/\tilde{D})]}{\frac{1}{\tilde{D}}\oint dy \oint dx\ e^{-\Psi(y)/\tilde{D}} e^{\Psi(x) - 2\pi\eta\Theta(y-x)}}, \tag{5}$$

where $\Theta(x)$ is the Heaviside step function. For $\eta(t)$ which changes on time scales much slower than the principle relaxation time of the system, the net voltage is found by averaging

$$\langle V\rangle = \frac{2e}{\hbar} \lim_{\tau\to\infty} \frac{1}{\tau} \int_0^{\tau} \left\langle \dot{\phi}(t) \right\rangle\ dt.$$

We have chosen this first example in the hopes that the mechanisim for the production of this net voltage will be clear to the average reader: namely, the voltage is due to a net bias in the "hopping" of the phase difference through an angle of $\pm 2\pi$. Since the effect is typically exponential in the applied force $f(t)$, a force with mean zero can give rise to a net voltage if the force is applied asymmetrically in time. Since $\langle\dot{\phi}\rangle$ is an antisymmetric nonlinear function of $\eta(t)$ *hs* and the voltage can be expanded in a series in the odd moments of $\eta(t)$ $\langle V\rangle = \sum_{n=1}^{\infty} c_{2n+1} \left\langle \eta^{2n+1}\right\rangle$. Therefore there will be a net voltage whenever any odd moment $\langle\eta^{2n+1}\rangle \neq 0$. This happens even though the net "force" is zero, and therefore is a fluctuation-induced effect. For nonadiabatic noise this statement has to be generalized to the statement that there will be a current if there is a nonvanishing odd order *correlation function* of the noise.

The net voltage in Eq. (5) can be evaluated by steepest descents when $D, \eta \ll 2\omega_j$. The result is

$$\langle V(t)\rangle = \left(\frac{8e^2RJ_c}{\hbar^2}\right) e^{-J_c\hbar/eRD} \sinh\left(\frac{\pi\hbar f(t)}{2eRD}\right)\ .$$

As we have mentioned, the averaged voltage can be expanded in terms of the odd moments of $f(t)$,

$$\langle V\rangle = \left(\frac{8e^2RJ_c}{\hbar^2}\right) e^{-J_c\hbar/eRD} \times \sum_{n=0}^{\infty} \left(\frac{\pi\hbar}{2eRD}\right)^{2n+1} \frac{\langle f^{2n+1}\rangle}{(2n+1)!}\ .$$

This asymmetry can also be viewed in terms of the spectral properties of the noise. Any noise with a uniform distribution of phases, such as Gaussian noise, will be symmetric, and will not give rise to a net voltage. *If the phase distribution of the spectrum is asymmetric the noise will give rise to a net voltage.* For arbitrarily fast noise the net voltage will be nonvanishing whenever any odd cumulant is nonvanishing, $\langle \eta^{m_1}(t_1), \ldots, \eta^{m_N}(t_n)\rangle \neq 0$, where $\sum_{i=1}^{n} m_i$ is odd. This phasic asymmetry, which we call *dynamical asymmetry*, gives rise to a number of interesting fluctuation-induced effects, and is worthy of further study.

Figure 2 shows that the current is a peaked function of the noise strength. Thus, everything else being constant, there is an optimal amplitude for the driving. Here the main features introduced by the dynamical asymmetry are the interplay of the lower potential barriers in the positive direction relative to the negative direction (for this particular driving) and for the corresponding shorter and longer times, respectively, the force is felt. These types of competitive effects appear ubiquitously in systems where there is an interplay between thermal activation and dynamics.

As our principle example we consider the only example of nontrivial noise that (to our knowledge) can be treated analytically, and can also exhibit temporal asymmetry. This is a system which is driven by telegraph $I_n(t)$ (dichotomous) noise

$$\dot{\phi} + w_j \sin\phi = \eta(t)\ , \tag{6}$$

where $\eta(t) = (2eR/\hbar)I(t)$. The telegraph noise has two states,

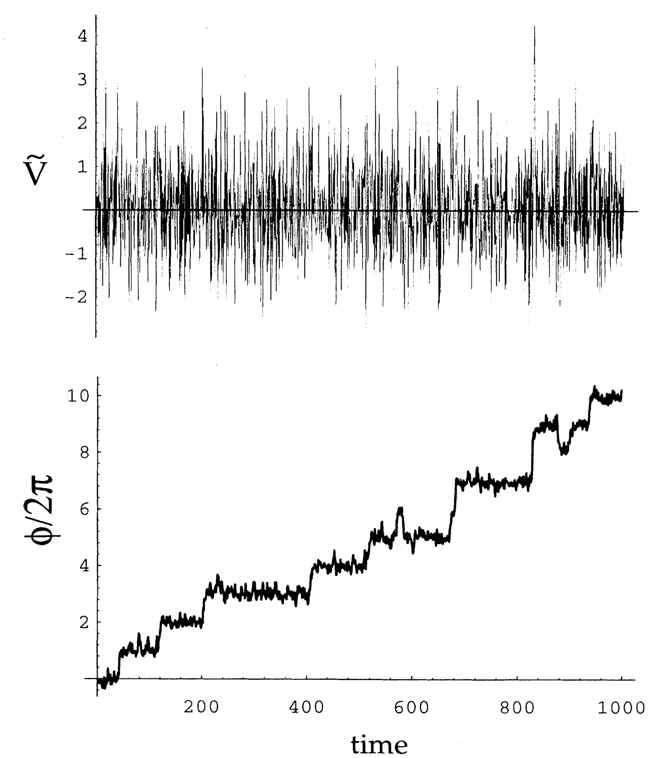

FIG. 2. Times series.

$$\eta_+ = \sqrt{\frac{D}{\tau}\left(\frac{1+\epsilon}{1-\epsilon}\right)}, \quad \eta_- = -\sqrt{\frac{D}{\tau}\left(\frac{1-\epsilon}{1+\epsilon}\right)}. \tag{7}$$

The transition probabilities w_+ from the plus to the minus state and w_- from the minus to the plus state are

$$w_+ = \frac{1+\epsilon}{2\tau}, \quad w_- = \frac{1-\epsilon}{2\tau}. \tag{8}$$

This noise has mean zero $\langle \eta(t) \rangle = 0$ and correlation function

$$\phi(t) = \langle \eta(t)\eta(0) \rangle = \frac{D}{\tau}\exp\left(-t/\tau\right). \tag{9}$$

There are a number of stochastic processes with precisely this correlation function, but not all of them will give rise to a net voltage in the junction. This noise is dynamically asymmetric in the same sense as our first example. The noise spends a different amount of time in each state on the average, yet the the average force is still zero.

The system is described by the set of equations [13]

$$\partial_t \rho_+ = -\partial_x[\omega_j \sin\phi + \eta_+]\rho_+ - w_+\rho_+ + w_-\rho_- \ , \tag{10}$$

$$\partial_t \rho_- = -\partial_x[\omega_j \sin\phi + \eta_-]\rho_- - w_-\rho_- + w_+\rho_+ \ , \tag{11}$$

where $\rho = \rho_+ + \rho_-$. The net voltage can easily be found [15]

$$\langle V \rangle = (4\pi e/\hbar)\mu(\phi)[\omega_j \sin\phi - D\partial_\phi W(\phi)]\rho_s(\phi). \tag{12}$$

$$\mu(\phi) = [1 + \tau\omega_j\cos\phi]^{-1}, \quad \theta = 2\epsilon/\sqrt{1-\epsilon^2}$$

$$W(x) = \left[1 - (\tau/D)\omega_j^2\sin^2\phi(x) - \theta\sqrt{\tau/D}\omega_j\sin\phi\right].$$

Again imposing periodic boundary conditions and normalizing we obtain

$$\langle V \rangle = \left(\frac{4\pi e}{\hbar}\right)\frac{1-\exp(\Delta/D)}{N}, \quad \Delta = \oint dy \frac{\omega_j \sin y}{W(y)} \tag{13}$$

$$\Psi(x) = -\int^x dy \frac{\omega_j \sin y}{W(y)} \ , \tag{14}$$

$$N = \frac{1}{D}\oint dy \oint dx \frac{e^{-\Psi(y)/D} e^{\Psi(x) - \Delta\Theta(y-x)}}{\mu(x)W(y)} \ . \tag{15}$$

To order $\sqrt{\tau/D}$ we have

$$\langle V \rangle = \left(\frac{2e\omega_j^2}{\hbar}\right)\frac{\epsilon}{\sqrt{1-\epsilon^2}}\frac{\sqrt{\tau/D}}{I_0^2(w_j/D)} \ , \tag{16}$$

where I_0 is a modified Bessel function. The current is positive if $\epsilon > 0$ and negative if $\epsilon < 0$.

There are obviously an unlimited number of possible examples of dynamically asymmetric noise. The dichotomic noise used above is a special case of a multistate "Kubo-Anderson" process [14] (sometimes called the "kangaroo processes"). There are also types of continuous noise which can have dynamical asymmetry. Shot noise, which is also of great importance in quantum electronics, is of this latter type. Mean zero shot noise, which is temporally asymmetric, can be produced if the frequency and amplitude distribution is slightly different for positive and negative fundamental pulses.

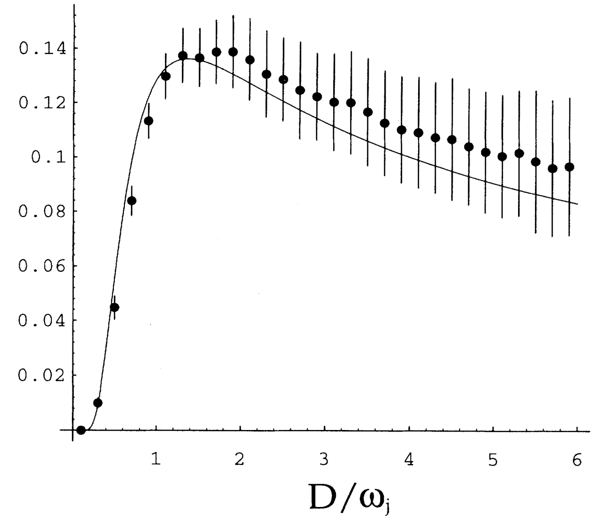

FIG. 3. Mean voltage $\langle V \rangle$ vs noise strength D for $\tau = 0.01$ and $\epsilon = 0.9$. The solid line is the theoretical prediction and the dots and error bars are the results from numerical simulations.

While in the previous cases the energy was extracted in the form of a net transport of particles, here the nonequilibrium effects of the driving are translated into the more generic form of a net voltage. In previous work only dynamically symmetric noise was examined, and the effect vanished in any symmetric situation, as is the case with Josephson junctions. Since many periodic structures in condensed matter physics are also symmetric, we feel that our observations here might have significant practical im-

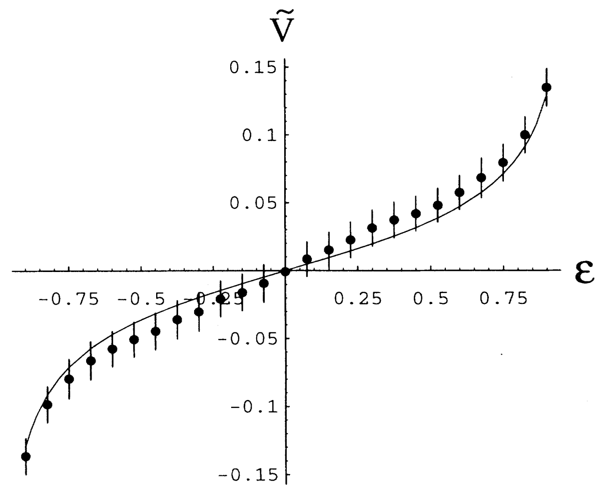

FIG. 4. Mean voltage $\langle V \rangle$ vs temporal asymmetry parameter ϵ for $\tau = 0.01$ and $D = 2.0$. The solid line is the theoretical prediction and the dots and error bars are the results from numerical simulations.

plication. For example, dynamically asymmetry voltage fluctuations should give rise to net currents in superionic conductors. These systems typically have periodic and symmetric internal fields, and the effect would be precisely analogous to the situation discussed here.

Dynamical asymmetry and spatial asymmetry relate to the problem of nonequilibrium transport in *precisely the same way*. In both cases a net effect arises due to an interplay between the strength of a fluctuation, the time it acts, and underlying dynamics. In the case of a spatial asymmetry a fluctuation to the right with a given strength which lasts a given time will tend to take the system over the right-hand barrier while the same fluctuation with sign reversed does not lift the system over the left-hand barrier. In the case of dynamical asymmetry the probabilities of the fluctuations to the right and left are different, so a net effect arises in the absence of a spatial asymmetry. What both of them show is that even a subtle asymmetry in the *shape* of the potential or in the *shape* of the spectral properties of the noise will give rise to an effect even when the net force due to each vanishes. Previously all the emphasis has been put on spatial asymmetry, but we believe that dynamical asymmetry deserves to be put on equal footing with spatial asymmetry as one of the principle elements which, in combination with the nonequilibrium time correlations, can give rise to a net effect.

At this point it appears that the basic principles behind fluctuation-induced transport type phenomena in overdamped Ohmic systems are well understood. The lesson to be learned from all this is that any kind of broken symmetry will usually allow energy to be pumped out of a "bath" as long as the system as a whole is not in equilibrium [16]. This occurs at the expense of the increased entropy of the bath, and is the essence of the "surprising" effects observed in [2–5]. From the standpoint of fundamental physics this observation is the main significance of an understanding of fluctuation-induced transport.

From the standpoint of applications, these ideas may be of significant practical value for manipulating molecule sized objects which are subject to thermal fluctuations because of their small size. The attractiveness of this idea is based largely on the observation that thermal fluctuations can, under the right nonequilibrium conditions, be harnessed to perform useful work. Thus, thermal fluctuations can be made to aid the manipulation, rather than hinder it. Such methods have been proposed as novel molecular separation techniques [5]. More recently, we have shown that it should be theoretically possible to control the molecular motions of voltage sensitive biomolecules by applying voltage fluctuations of a type very similar to the ones discussed in this paper [8]. This can be done to such a degree that the molecule can essentially be placed in a desired kinetic substate by a process we have called nonequilibrium kinetic focusing.

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